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AXISYMMETRIC ASCENDING FLOW IN GROUNDWATER PRODUCTION WELL OF MONZOUNGOUDO, BENIN

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Abstract— This paper aims to study the axisymmetric ascending flow in a groundwater production well from a captive reservoir of Monzoungoudo. In axisymmetric flow ascending, the pressure p is a fundamental hydrodynamic parameter which allows to calculate the behavior of the production water resource. The static pressure of the reservoir and therefore the analysis of groundwater pressure variation through the well are determinate parameters. This study proved that the water pressure decreases from the bottom to the head of the well. The pressure difference (discharge) between the reservoir and the wellhead is the sum of the losses of pressure in the reservoir and the pressure drop in the well.

Index Terms— Axisymmetric flow, captive reservoir, groundwater, hydrodynamic parameter, pressure, production well, MATLAB.

1 Introduction

TO improve the conditions of groundwater exploitation, we need a mathematical model to analyze the water flows in the production well. Water flows from the well bottom to the top of the well due to the pressure in the reservoir. The hydrodynamic conditions of the water depend of the characteristics of the production well. The measured data at each moment is the water pressure in the production well.

2 Position of the Problem

The problem to solve is to analyze the vertical flow through the tubing from the bottom of the production well to its head, by a mathematical model. The figure 1 shows this problem of pressure distribution along the well.

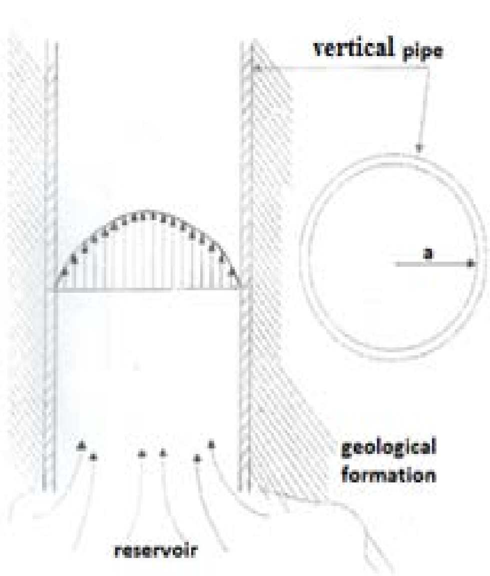

Fig 1. Representation of the problem [2]

3 Physical Model

The physical model shown in figure (2) constituted by a well producing water with rate Q , the well is tubing which radius is a with a flow of water from a captive reservoir in the deep zone. The medium is ascending according to up oriented vertical axis (0,z).

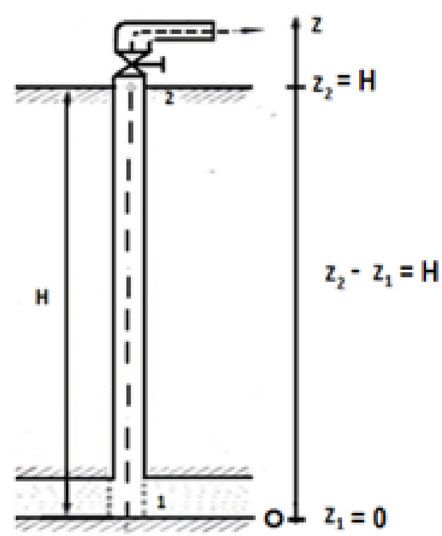

Fig 2. Position of the ascending flow through the tubing of the boring [4]

4 Materials and Method

For processing the data we used following process tools: the Bernoulli equation, the law of Darcy- Weisbach, the Colebrook-White formula, the Reynolds numbers to describe the mathematical model flow in the well. The matlab language is used, for solving the mathematical model flow in the well. For resolution of the governing equations, analytical method in adopted.

4.1 Data of the Problem

The characteristics of the porous medium are, permeability, porosity ϕ. The medium being assumed to be known its permeability k, his porosity ϕ, and the productive thickness e, the water its density ρ, dynamic viscosity μ and cinematic viscosity ϑ at 20°C, the tubing diameter d, the production well depth H, the flow rate Q, the roughness of the tubing ε and the head pressure p_2 are data of the problem.

4.2 Hydrogeogical Data

The hydrogeological data used during this study are mainly the technical data obtained during the drilling of the groundwater well (table1) and provided by the general center of hydraulic services (DGEau, May 2000)

The main hydrogeological characteristics are: transitivity, dynamic viscosity, kinematic viscosity, flow rate, density of the fluid.

TABLE 1
GROUNDWATER RESERVOIR AND WELL DATA

CHARACTERISTICS OF RESERVOIR AND WELL	
Parameters	Values
Permeability k (Darcy)	21
Depth of the well H (m)	244,18
Thickness of the reservoir e (cm)	4318
Diameter of the well d (m)	0,126
Flow rate Q (cm^3/s)	2000
Density of the fluid ρ (Kg/m^3)	1000
Acceleration of the gravity g (m/S^2 ou N/Kg)	9,81
dynamic viscosity μ (Centipoise)	0,89
Kinematic viscosity ϑ (m^2/S)	0,89.10^{-6}
Pressure at the head of the well p_2 (bars)	4,16
Roughness of the pipe ε (mm)	0,12

5 Calculation of the Static Pressure in the Reservoir

The static pressure of the fluid in the reservoir is defined, according to the equation of Bernoulli, and Codo, F.P. and al. (2012) [5], at the head of the well, the pressure is:

$$p_2 = p_1 - \rho g H - \lambda \rho H \frac{v^2}{2d} \qquad (1)$$

After transformations:

$$p_2 = p_1 - \rho g H - \frac{\mu Q}{2\pi e k} \, ln\left(\frac{R}{a}\right) - 0{,}06642\rho \frac{H}{d^{4,8}} Q^{1,8} \nu^{0,2} \qquad (2)$$

With $ln\left(\frac{R}{a}\right) = 2\pi$, we

With $R_e = \frac{v\,d}{\vartheta}$ and $v = \frac{4Q}{\pi d^2}$

$$p_{st} = p_2 + \rho g H + \frac{\mu Q}{ek} + 0{,}06642\rho \frac{H}{d^{4,8}} Q^{1,8} \nu^{0,2} \qquad (3)$$

6 Analytical Resolution of the Problem

6.1 Boundary Conditions

The boundary conditions taken into account for the resolution of the mathematical modeling and the simplifying assumptions, are the following;

- the fluid is considered as incompressible $(\rho = cste)$,
- the fluid is Newtonian $(\mu = cste)$,
- the flow is steady $(\frac{\partial}{\partial t} = 0)$, ,
-the flow is axisymmetric, the changes in quantities in the azimuthal direction did not exist $(\frac{\partial}{\partial \theta} = 0)$ and $v_\theta = 0$

6.2 Governing Equations

6.2.1 The explicit calculation of the friction coefficient λ in circular pipe

We are interested in the modeling of an incompressible flow in an axisymmetric pipe. This flow is turbulent and involves some flow phenomena that need to be analyzed correctly. Among these phenomena, the pressure drop can influence the nature of the flow in the field of study. The chosen model therefore requires the writing of an equation governing the pressure arising from the principle of conservation of energy. The modeling of this flow is described by mathematical relations between the altitude z, the velocity of flow v and the pressure p in a frame (o, z). It has been possible through the use of the Bernoulli formula, the Colebrook-White formula, the Weisbach formula, the conservation equation, which can be written as follows:

$$\frac{v_1^2}{2g} + \frac{p_1}{\rho g} + z_1 = \frac{v_2^2}{2g} + \frac{p_2}{\rho g} + z_2 + \Delta h \qquad (4)$$

$$\Delta h = \lambda \frac{H}{d} \frac{v^2}{2g} \qquad (5)$$

$$\frac{1}{\sqrt{\lambda}} = -2\log\left(\frac{2{,}51}{Re\sqrt{\lambda}} + \frac{\varepsilon}{3{,}7d}\right) \qquad (6)$$

The Determination of the coefficient of friction is important in the calculation of the pressure loss Δh. The formula used generally, is the Colebrook-White (6), an implicit formula, with the Moody diagram.

Achour and Bedjaoui (2006) [1] are intended for the explicit calculation of the friction coefficient λ in circular pipe for evaluating the gradient Δh of the drop. The formula proposed by Achour and Bedjaoui (2006) [1] is the exact solution for Colebrook-White equation (6). The coefficient of friction λ is expressed in the following explicit form:

$$\lambda = \left[-2log\left(\frac{\varepsilon/d}{3{,}7} + \frac{10{,}04}{\bar{R}}\right)\right]^{-2} \qquad (7)$$

The parameter R appearing in the relationship (7) represent the Reynolds number characterizing the flow in a reference rough model. The relationship exact of R has not yet been established, but an approximate relationship was however proposed by

Achour et Bedjaoui (2006) [1], which shows that by R is a function of the relative roughness ε/d and the number of Reynolds *Re* characterizing the flow in line considered :

$$\bar{R} = 2R_e\left[-log\left(\frac{\varepsilon/d}{3{,}7}+\frac{5{,}5}{{R_e}^{0{,}9}}\right)\right]^{-1} \qquad (8)$$

And when the relative roughness ε/d and the number of Reynolds R_e are the known parameters of the problem, the simultaneous use of relations (7) and (8) provides explicitly assess the coefficient of friction λ. The relationships (7) and (8) are applicable in the entire field of turbulent flow and cover the entire Moody diagram. In this paper, we propose to test this formula, and thus to prove its veracity. We consider a fluid flowing in a circular pipe characterized by a Reynolds number $R_e = 2.10^6$ for a relative roughness $\varepsilon/d = 5.10^{-4}$. To calculate the coefficient of friction with the explicit formula (7), the R parameter is calculate according to formula (8):

$$\bar{R} = 2\times 2.10^6\left[-log\left(\frac{5.10^{-4}}{3{,}7}+\frac{5{,}5}{(2.10^6)^{0{,}9}}\right)\right]^{-1} = 1043413{,}87$$

The value of the coefficient of friction λ, according to equation (7) is equal to:

$$\lambda_{Achour/Bedjaoui} = \left[-2log\left(\frac{5.10^{-4}}{3{,}7}+\frac{10{,}04}{1043413{,}87}\right)\right]^{-2} = 0{,}0169494$$

In substituting this calculated value of the coefficient of friction in the equation (6) we obtain:

$$\frac{1}{\sqrt{\lambda_{Colebrook}}} = -2\log\left(\frac{2{,}51}{2.10^6\sqrt{0{,}0169494}}+\frac{5.10^{-4}}{3{,}71}\right) = 7{,}681$$

And we get $\lambda_{Colebrook} = 0{,}0169498$
We have

$$\left|\lambda_{Colebrook} - \lambda_{Achour/Bedjaoui}\right| < 10^{-4}$$

The results obtained with the formulas (6) and (7) show that the error is 10-6 which confirms the formula proposed by Achour and Bedjaoui (2006) [1].
The formula of the coefficient of friction becomes:

$$\lambda = \left[-2log\left(\frac{\varepsilon/d}{3{,}71}+\frac{10{,}04}{2R_e\left[-log\left(\frac{\varepsilon/d}{3{,}71}+\frac{5{,}5}{{R_e}^{0{,}9}}\right)\right]^{-1}}\right)\right]^{-2} \qquad (9)$$

$$R_e = \frac{v\,d}{\vartheta} \quad et \quad v = \frac{4Q}{\pi d^2}$$

With reference to formula (5), we obtain:

$$\Delta h = \left[-2log\left(\frac{\varepsilon/d}{3{,}71}+\frac{3{,}941\,d\,\vartheta}{Q\left[-log\left(\frac{\varepsilon/d}{3{,}71}+4{,}423\frac{d^{0{,}9}\vartheta^{0{,}9}}{Q^{0{,}9}}\right)\right]^{-1}}\right)\right]^{-2}\frac{H}{d}\frac{16Q^2}{2\pi^2 d^4 g}$$

$$\Delta h = 0{,}811\frac{H.Q^2}{gd^5}\left[-2log\left(\frac{\varepsilon/d}{3{,}71}+\frac{3{,}941\,d\,\vartheta}{Q\left[-log\left(\frac{\varepsilon/d}{3{,}71}+4{,}423\frac{d^{0{,}9}\vartheta^{0{,}9}}{Q^{0{,}9}}\right)\right]^{-1}}\right)\right]^{-2} \qquad (10)$$

Since there is mass conservation and the fluid being incompressible we have:

Q=S∗v=constant (11)

Since the input and output section of the domain is assumed to be identical, then the velocity is constant in the pipe. However, it cannot be considered that the sections are identical until the entry of each section. We can write the velocity at the entrance and exit to the well are equal:

$$v_1 = v_2 \qquad (12)$$

6.2.2 The determination of the pressure in function of the depth z

According to fig.2 we have $H = z_2 - z_1$ and considering the relationships of (4) to (12), we obtain equation (4) which written:

$$\frac{p_1}{\rho g} - H = \frac{p_2}{\rho g} + 0{,}811\frac{H.Q^2}{g\,d^5}\left[-2log\left(\frac{\varepsilon/d}{3{,}71}+\frac{3{,}941\,d\,\vartheta}{Q\left[-log\left(\frac{\varepsilon/d}{3{,}71}+4{,}423\frac{d^{0{,}9}\vartheta^{0{,}9}}{Q^{0{,}9}}\right)\right]^{-1}}\right)\right]^{-2} \qquad (13)$$

$$p_2 = p_1 - \rho g H - 0{,}811\frac{H.\rho.Q^2}{d^5}\left[-2log\left(\frac{\varepsilon/d}{3{,}71}+\frac{3{,}941\,d\,\vartheta}{Q\left[-log\left(\frac{\varepsilon/d}{3{,}71}+4{,}423\frac{d^{0{,}9}\vartheta^{0{,}9}}{Q^{0{,}9}}\right)\right]^{-1}}\right)\right]^{-2} \qquad (14)$$

$$p_2 = p_{st} - \frac{\mu Q}{2\pi e\,k}\ln\left(\frac{R}{a}\right) - \rho g H - 0{,}811\frac{H.\rho.Q^2}{d^5}\left[-2log\left(\frac{\varepsilon/d}{3{,}71}+\frac{3{,}941\,d\,\vartheta}{Q\left[-log\left(\frac{\varepsilon/d}{3{,}71}+4{,}423\frac{d^{0{,}9}\vartheta^{0{,}9}}{Q^{0{,}9}}\right)\right]^{-1}}\right)\right]^{-2} \qquad (15)$$

By making the approximation Laurent, H., Fabris, H. and Gringarten, C. (1972): $ln(\frac{R}{a}) = 2\pi$

$$p_2 = p_{st} - \frac{\mu Q}{e\,k} - \rho g H - 0{,}811\frac{H.\rho.Q^2}{d^5}\left[-2log\left(\frac{\varepsilon/d}{3{,}71}+\frac{3{,}941\,d\,\vartheta}{Q\left[-log\left(\frac{\varepsilon/d}{3{,}71}+4{,}423\frac{d^{0{,}9}\vartheta^{0{,}9}}{Q^{0{,}9}}\right)\right]^{-1}}\right)\right]^{-2} \qquad (16)$$

Taking into account of the variable z, we can write the pressure at the altitude z:

$$p(z) = p_{st} - \frac{\mu Q}{e\,k} - \rho g z - 0{,}811\frac{z.\rho.Q^2}{d^5}\left[-2log\left(\frac{\varepsilon/d}{3{,}71}+\frac{3{,}941\,d\,\vartheta}{Q\left[-log\left(\frac{\varepsilon/d}{3{,}71}+4{,}423\frac{d^{0{,}9}\vartheta^{0{,}9}}{Q^{0{,}9}}\right)\right]^{-1}}\right)\right]^{-2} \qquad (17)$$

6.2.2 The determination of the pressure in function of flow rate Q

Taking into account of the variation of the produced flow rate, we can express the pressure as follows:

$$p(Q) = p_{st} - \frac{\mu Q}{e\,k} - \rho g H - 0{,}811\frac{H.\rho.Q^2}{d^5}\left[-2log\left(\frac{\varepsilon/d}{3{,}71}+\frac{3{,}941\,d\,\vartheta}{Q\left[-log\left(\frac{\varepsilon/d}{3{,}71}+4{,}423\frac{d^{0{,}9}\vartheta^{0{,}9}}{Q^{0{,}9}}\right)\right]^{-1}}\right)\right]^{-2} \qquad (18)$$

The value of the pressure $p(Q)$, according to equation (3) is equal to:

$$p(Q) = p_{st} + 0{,}06642\rho\frac{H}{d^{4.8}}Q^{1.8}\nu^{0.2} - 0{,}811\frac{H.\rho.Q^2}{d^5}\left[-2log\left(\frac{\varepsilon/d}{3{,}71}+\frac{3{,}941\,d\,\vartheta}{Q\left[-log\left(\frac{\varepsilon/d}{3{,}71}+4{,}423\frac{d^{0{,}9}\vartheta^{0{,}9}}{Q^{0{,}9}}\right)\right]^{-1}}\right)\right]^{-2} \qquad (19)$$

7 Results and Discussions

The resolution of the equation (17) and (19) is made using the Matlab language. The results obtained are shown in Table 2 and the table 3.

TABLE 2

PRESSURE VARIATION IN THE WELL AS A FUNCTION OF THE DEPTH IN THE LOCALITY OF MONZOUNGOUDO

RESULT OF THE CALCULATION OF THE PRESSURE IN THE WELL $p(z)$:													
z (m)	0	20	40	60	80	100	120	140	160	180	200	220	244,18
$p(z)$(bars)	28,12	26,16	24,19	22,23	20,27	18,31	16,34	14,38	12,42	10,46	8,50	6,54	4,16

TABLE 3

PRESSURE VARIATION AT THE HEAD OF THE WELL AS A FUNCTION OF THE FLOW RATE IN THE LOCALITY OF MONZOUNGOUDO

RESULT OF THE CALCULATION OF THE PRESSURE IN THE PRODUCTION $p(Q)$									
Q (m³/s)	0,002	0,003	0,004	0,005	0,006	0,007	0,008	0,009	0,01
$p(Q)$(bars)	4,161	4,163	4.165	4.167	4.170	4,173	4,176	4,180	4,184

The variation of the pressure $p(z)$ in the well as a function of the depth z and the variation of the pressure $p(Q)$ in the production as a function of the flow rate Q are shown respectly in fig. 3 and in fig. 4.

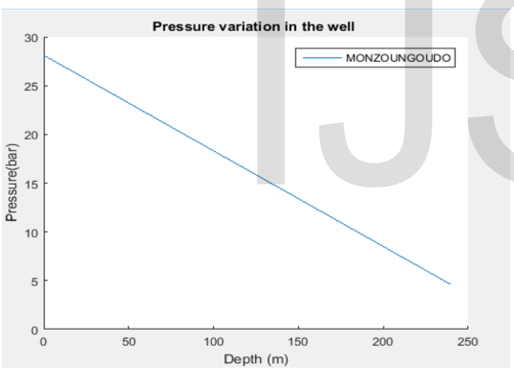

Fig. 3. Pressure variation in the well as a function of the depth in the locality of Monzoungoudo

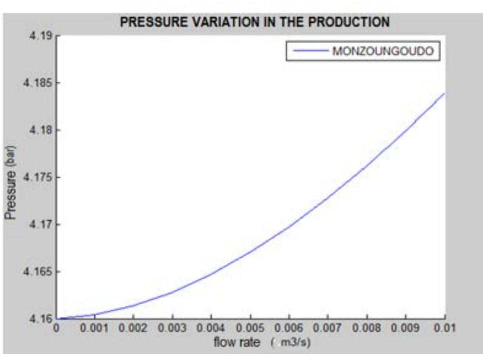

Fig. 4. Pressure variation in the well as a function of the flow rate in the locality of Monzoungoudo

The figure 3 and the figure 4 make it possible to evaluate respectly the variation in the pressure in the well as a function of the depth and the variation in the pressure in the production as a function of the flow rate at the experimental site of Monzoungoudo. It can be seen that the values of the pressures decrease as one approaches the surface of the soil to take the value of 4, 16 bars at the wellhead. It can be seen also that the values of the pressures in the production increases in function of the flow rate.

8 CONCLUSION

The simulation of the flows made it possible to demonstrate the existence of a pressure gradient that varies as a function of the depth of the well. The effect of pressure gradient is unavoidable since there exist in the production pipes of the asperities due to the manufacturing defect, thus causing pressure drops during the flow of the fluid.

REFERENCES

[1] B. Achour., and A. Bedjaoui., 2006. Calcul du coefficient de frottement en conduit circulaire sous pression. Larhyss Journal, ISSN 1112-3680, no05, Juin 2006, pp.197-200.

[2] Ben, L.M., 2002. Simulation des écoulements dans les puits producteurs avec injection. Mémoire de fin d'étude d'Ingénieur d'Etat, Université Ouargla d'Algérie, 77p.

[3] Berni, N et O.E. Yacoub., 2012. Etude numérique d'un écoulement dans un milieu poreux. Mémoire de Master Professionnel, Université Kasdi Merbah Ouargla d'Algérie,

[4] Codo, F.P., 1989. Etude Analytique des coactions hydrodynamiques et thermodynamique entre les réservoirs, les puits de production et de Réinjection Géothermiques. Thèse de Doctorat ès Sciences Minières, Université Polytechnique de l'Industrie Lourde Miskolc, Hongrie, 127p.

[5] Codo, F.P., V, Adanhounmè., and A, Adomou., 2012.Analytical approach for the determination of complex potential and pressure in the production and reinjection wells of geothermal reservoir. Journal of Applied Sciences Research, 8(1): 261-266.

[6] E. Rieutord., 1985. Ecoulement stationnaire en conduite de section constante d'un fluide visqueux isochore – Perte de charge. INSA de Lyon, 23p.

[7] Laurent, H.,Fabris et A.C. Gringarten., 1972. Etude des possibilités d'exploitation de l'eau chaude du DOGGER dans la région du CORBEIL ESSONNES (91). Bureau de Recherches Géologiques et Minières, Département de Géothermie, Orléans, 41p.